

Flavor structure of E_6 GUT models

Hidetoshi Kawase ^{*}, Nobuhiro Maekawa [†]

Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Abstract

It has been pointed out that in E_6 grand unified theory with $SU(2)_H$ family symmetry, the spontaneous CP violation can solve the supersymmetric CP problem. The scenario predicts $V_{ub} \sim \mathcal{O}(\lambda^4)$ rather than $\mathcal{O}(\lambda^3)$ which is naively expected value, because of a cancellation at the leading order. Since the experimental value of V_{ub} is $\mathcal{O}(\lambda^4)$, it must be important to consider the reason and the conditions for the cancellation. In this paper, we give a simple reason for the cancellation and show that in some E_6 models such a cancellation requires that the vacuum expectation value (VEV) of the adjoint Higgs does not break $U(1)_{B-L}$. Note that this direction of the VEV plays an important role in solving the doublet-triplet splitting problem by Dimopoulos-Wilczek mechanism. In this E_6 models, the experiments may measure the direction of the adjoint Higgs VEV by measuring the size of $V_{ub} \sim \mathcal{O}(\lambda^4)$.

^{*}e-mail: hkawase@eken.phys.nagoya-u.ac.jp

[†]e-mail: maekawa@eken.phys.nagoya-u.ac.jp

1 Introduction

Grand unified theory (GUT) has several attractive features [1, 2]. It unifies not only three forces but also matter (quark and lepton) fields in the standard model (SM). As the result of the unification, the origin of the hypercharge, $U(1)_Y$, in the SM can be understood in the GUT. The unification of three gauge interactions has quantitatively been tested by calculating three running gauge couplings in the (supersymmetric) SM which meet at a scale (the GUT scale). The unification of matter fields roughly explains the various hierarchical structures of quark and lepton masses and mixings by an simple assumption that the **10** multiplets of $SU(5)$ induce the hierarchical structures of Yukawa couplings. Actually, in $SU(5)$ unification let us assume that the Yukawa couplings which include the first two generation of **10** multiplets, **10**₁ and **10**₂ have suppression factors, ϵ_1 and ϵ_2 ($\epsilon_1 \ll \epsilon_2 \ll 1$), respectively. Then, we can understand not only that the mass hierarchy of up quarks is the strongest, that of neutrinos is the weakest and those of the down quarks and charged leptons are in between those of up quarks and neutrinos, but also that the quark mixings are smaller than the neutrino mixings.

One of the most attractive features of the E_6 unification [3, 4, 5, 6, 7, 8, 9] is that the assumption for the Yukawa hierarchies can be derived in a reasonable setup [10, 11, 12]. The derivation is so natural that even if we introduce a family symmetry, $SU(2)_H$ or $SU(3)_H$, for unifying the first two generations or all three generations into a single multiplet, (**27**, **2**) or (**27**, **3**), realistic Yukawa matrices can be easily obtained by the spontaneous breaking of the family symmetry [13, 11, 12]. As the result of the unification of generations, SUSY flavor problem [14, 15] can be solved even if large neutrino mixings are realized.

Recently, it has been pointed out that in the E_6 unification, if CP symmetry is spontaneously broken [16, 17, 18, 19] by the vacuum expectation value which breaks the family symmetry [20, 21, 22, 23], the Kobayashi-Maskawa (KM) phase [24] can be induced while the SUSY CP problem is solved [25, 26]. Generically, in the non-universal sfermion masses which are predicted by the family symmetry, complex Yukawa couplings, which are required to produce non-vanishing KM phase, induce complex off-diagonal sfermion masses which result in too large electric dipole moments (EDM) of the neutron [27]. It is non-trivial that this problem can be solved by this spontaneous CP violation. Moreover, it has been pointed out that in the model [26], the (1, 3) component of the KM matrix, V_{ub} , becomes rather smaller value, $\mathcal{O}(\lambda^4)$, than the expected value, $\mathcal{O}(\lambda^3)$, because of a cancellation. Here, $\lambda \sim 0.22$ is the Cabibbo angle [28]. Since the experimental value of V_{ub} is of order λ^4 rather than of order λ^3 [29], it must be important to study the reason of this cancellation and to know the conditions for the cancellation in the E_6 GUT model.

In this paper, we examine the reason and the conditions for the cancellation. First, we show that in the E_6 GUT such cancellation is not special, i.e., the quark mixings, not only V_{ub} but also Cabibbo mixing, tend to be small because of the unification of the Yukawa couplings. Such cancellation due to Yukawa unification has been known in $SO(10)$ GUT, but even in E_6 GUT, it has been discussed in the literature [9]. To avoid this cancellation, the mixing of doublet Higgses and/or the higher dimensional interactions which include the E_6 Higgses (adjoint Higgs and fundamental Higgs) are required. It is not so easy that the effect of the higher dimensional interactions becomes sizable. One easy way is to introduce the anomalous $U(1)_A$ gauge symmetry [30, 31, 32, 33], with which the contribution of the higher dimensional interactions becomes the same order as that of the tree level interactions. Another way is to introduce some symmetry which forbids

the tree level interactions and allows the higher dimensional interactions. The family symmetry, $SU(2)_H$, can play the role, if the first two generation fields behave as doublet under the $SU(2)_H$. The $(1, 2)$ component is produced not from the tree level interactions but from the higher dimensional interactions which include the adjoint E_6 Higgs. Second, we consider the effect of the direction of the E_6 adjoint VEV in the E_6 model with the family symmetry. It is shown that the cancellation requires that the adjoint VEV does not break $U(1)_{B-L}$. Note that the direction of the VEV which remains $U(1)_{B-L}$ plays an important role in solving the doublet-triplet splitting problem by the Dimopoulos-Wilczek mechanism [34, 35, 36, 37, 38, 39].

This paper is organized as follows. In the next section, after a brief review for the E_6 model, we investigate the reason for the cancellation. In the section 3, we consider the effect of the direction of the adjoint Higgs VEV. In section 4, we give a summary.

2 Flavor structure of E_6 GUT models

In this section, after a brief review of E_6 GUT scenario, we show that in a simple setup in E_6 GUT, the Cabibbo mixing and V_{ub} are vanishing at leading order. And we discuss how to avoid the situation.

First, let us remind the basics of the E_6 group. The fundamental representation has 27 components and the adjoint representation has 78. These representations can be decomposed into the representation of $SO(10) \otimes U(1)_{V'}$ as

$$\mathbf{27} = \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4, \quad \mathbf{78} = \mathbf{45}_0 + \mathbf{16}_{-3} + \bar{\mathbf{16}}_3 + \mathbf{1}_0 \quad (2.1)$$

where the subscript indicates the $U(1)_{V'}$ charge of each representation. Similarly, $SO(10)$ representations can be decomposed in terms of $SU(5) \otimes U(1)_V$ representations:

$$\mathbf{16} = \mathbf{10}_{-1} + \bar{\mathbf{5}}_3 + \mathbf{1}_{-5}, \quad \mathbf{10} = \mathbf{5}_2 + \bar{\mathbf{5}}_{-2}, \quad \mathbf{45} = \mathbf{24}_0 + \mathbf{10}_4 + \bar{\mathbf{10}}_{-4} + \mathbf{1}_0. \quad (2.2)$$

The subscript means the $U(1)_V$ charge in this case.

Since we must break E_6 gauge group into the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \equiv G_{\text{SM}}$, we introduce two pairs of (anti-)fundamental fields Φ ($\bar{\Phi}$), C (\bar{C}) and an adjoint Higgs field A . Then we suppose that the vacuum expectation value (VEV) of Φ ($\bar{\Phi}$) breaks E_6 into $SO(10)$ and C (\bar{C}) breaks $SO(10)$ into $SU(5)$:

$$\langle \Phi \rangle = \langle \mathbf{1}_\Phi \rangle \neq 0, \quad \langle C \rangle = \langle \mathbf{16}_C [\ni \mathbf{1}_C] \rangle \neq 0. \quad (2.3)$$

In this paper, we often use a notation that the dimension of each field is expressed by its name. For example, $\mathbf{27}_\Phi = \mathbf{16}_\Phi + \mathbf{10}_\Phi + \mathbf{1}_\Phi$ etc. Here $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle|$ and $|\langle C \rangle| = |\langle \bar{C} \rangle|$ should be satisfied from the D -flatness conditions. On the other hand, since an adjoint Higgs A must break $SU(5)$ into G_{SM} , the VEV of the adjoint Higgs $\langle A \rangle$ can be generally written as

$$\langle A \rangle = xQ_{V'} + yQ_V + zQ_Y \quad (2.4)$$

where Q_X ($X = V', V, Y$) stands for a generator corresponding to the $U(1)_X$ charge. Note that $z \neq 0$ is needed to break $SU(5)$.

The matter fields in the standard model can be embedded in the fundamental representations Ψ_i with $i = 1, 2, 3$. Each generation of Ψ_i contains $\mathbf{10}$ and $\bar{\mathbf{5}}$ representations of

$SU(5)$:

$$\Psi_i = \mathbf{16}_i[\mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i] + \mathbf{10}_i[\bar{\mathbf{5}}_i + \bar{\mathbf{5}}'_i] + \mathbf{1}_i[\mathbf{1}'_i]. \quad (i = 1, 2, 3) \quad (2.5)$$

To obtain the Yukawa couplings, we have to fix the origin of the SM doublet Higgses. Here, for economical reason, we assume that the minimal supersymmetric SM (MSSM) Higgses come from Φ and C . (It is natural that one of the MSSM Higgses come from Φ if we consider how to realize doublet-triplet splitting naturally. But here, we do not address about it.) Then, we can obtain the Yukawa interactions from trilinear terms of $\mathbf{27}$ fields:

$$W = (Y_\Phi)_{ij}\Psi_i\Psi_j\Phi + (Y_C)_{ij}\Psi_i\Psi_jC. \quad (2.6)$$

Note that there are six $\bar{\mathbf{5}}$ in three $\mathbf{27}$. Three of six $\bar{\mathbf{5}}$ becomes superheavy with three $\mathbf{5}$ after breaking E_6 into $SU(5)$ through the above Yukawa interactions. The mass term between $\mathbf{5}_i$ and $\bar{\mathbf{5}}_i, \bar{\mathbf{5}}'_i$ fields can be expressed as $W_{\text{eff}} \supset \mathbf{5}M(\bar{\mathbf{5}}', \bar{\mathbf{5}})^\top$ where

$$M \equiv (Y_\Phi, rY_C) \langle \Phi \rangle, \quad r \equiv \langle C \rangle / \langle \Phi \rangle. \quad (2.7)$$

This means that the pairs of $\mathbf{5}$ and $\bar{\mathbf{5}}$ fields which obtain GUT scale masses are decoupled at low energy, and the massless modes are linear combinations of $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}'_i$. If higher generation fields have larger Yukawa couplings, then two $\bar{\mathbf{5}}$ fields from the third generation field $\mathbf{27}_3$ become superheavy unless $r \ll 1$. Therefore, it is reasonable that all three SM $\bar{\mathbf{5}}$ fields come from the first two generation fields, they have only smaller Yukawa couplings and milder hierarchical structures for the Yukawa couplings of $\bar{\mathbf{5}}$ fields. This is an important point to produce realistic quark and lepton masses and mixings in E_6 GUT. The mixing of $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}'_i$ for these massless modes are determined from the diagonalizing procedure

$$V^\top MU = \left(\begin{array}{ccc|c} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ \hline & & & \mathbf{0}_{3 \times 3} \end{array} \right) \quad (2.8)$$

where V and U are unitary matrices which rotate $\mathbf{5}_i$ and $(\bar{\mathbf{5}}_i, \bar{\mathbf{5}}'_i)$ into their mass eigenstates, respectively. Provided that $m_i \neq 0$ ($i = 1, 2, 3$), there remains three massless modes of $\bar{\mathbf{5}}$ fields and their mixing can be expressed as

$$\begin{pmatrix} \bar{\mathbf{5}}'_1 \\ \bar{\mathbf{5}}'_2 \\ \bar{\mathbf{5}}'_3 \end{pmatrix} = U_{\mathbf{10}}^0 \begin{pmatrix} \bar{\mathbf{5}}_1^0 \\ \bar{\mathbf{5}}_2^0 \\ \bar{\mathbf{5}}_3^0 \end{pmatrix} + U_{\mathbf{10}}^H \begin{pmatrix} \bar{\mathbf{5}}_1^H \\ \bar{\mathbf{5}}_2^H \\ \bar{\mathbf{5}}_3^H \end{pmatrix}, \quad \begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \end{pmatrix} = U_{\mathbf{16}}^0 \begin{pmatrix} \bar{\mathbf{5}}_1^0 \\ \bar{\mathbf{5}}_2^0 \\ \bar{\mathbf{5}}_3^0 \end{pmatrix} + U_{\mathbf{16}}^H \begin{pmatrix} \bar{\mathbf{5}}_1^H \\ \bar{\mathbf{5}}_2^H \\ \bar{\mathbf{5}}_3^H \end{pmatrix} \quad (2.9)$$

where $\bar{\mathbf{5}}_i^0$ and $\bar{\mathbf{5}}_i^H$ represent massless modes and heavy modes, respectively. The 3×3 mixing matrices $U_{\mathbf{10}}^0, U_{\mathbf{10}}^H, U_{\mathbf{16}}^0, U_{\mathbf{16}}^H$ are related with the unitary matrix U which appears in Eq. (2.8) so that

$$U \equiv \left(\begin{array}{c|c} U_{\mathbf{10}}^H & U_{\mathbf{10}}^0 \\ \hline U_{\mathbf{16}}^H & U_{\mathbf{16}}^0 \end{array} \right). \quad (2.10)$$

Note that by solving the following equation,

$$M \begin{pmatrix} U_{\mathbf{10}}^0 \\ U_{\mathbf{16}}^0 \end{pmatrix} = (Y_\Phi U_{\mathbf{10}}^0 + rY_C U_{\mathbf{16}}^0) \langle \Phi \rangle = 0 \quad (2.11)$$

$U_{\mathbf{10}}^0$ and $U_{\mathbf{16}}^0$ can be determined up to the multiplication of a 3×3 unitary matrix from the right. Because of the unitarity condition of U , three 6 dimensional vectors $(u_i)_k$ ($i = 1, 2, 3, k = 1, 2, \dots, 6$), which are defined as $(u_i)_j = (U_{\mathbf{10}}^0)_{ij}, (u_i)_{j+3} = (U_{\mathbf{16}}^0)_{ij}$, are

normalized and orthogonal. When the rank of Y_Φ equals to three, it is obvious that three independent vectors satisfied with Eq. (2.11) can be obtained by $(u_i)_{j+3} = \delta_{ij}$ and $(u_i)_j = r(Y_\Phi^{-1}Y_C)_{ij}$. Once we fix the vector space which satisfies Eq. (2.11), we can determine the explicit normalized and orthogonal basis for the space at any accuracy. Then the MSSM Yukawa interaction terms $(Y_u)_{ij}Q_i\bar{u}_jH_u$, $(Y_d)_{ij}Q_i\bar{d}_jH_d$ and $(Y_e^\top)_{ij}L_i\bar{e}_jH_d$ are obtained through

$$Y_u : \mathbf{10}_i\mathbf{10}_j\mathbf{5}_{H_u}, \quad (2.12)$$

$$Y_d, Y_e^\top : \mathbf{10}_i\mathbf{5}_j\mathbf{5}'_{H_d} + \mathbf{10}_i\mathbf{5}'_j\mathbf{5}_{H_d} \rightarrow \mathbf{10}_i(U_{16}^0\mathbf{5}^0)_j\mathbf{5}'_{H_d} + \mathbf{10}_i(U_{10}^0\mathbf{5}^0)_j\mathbf{5}_{H_d}. \quad (2.13)$$

Now let us examine the feature of quark mixings in this model. First, we consider a simple case where the MSSM Higgs fields H_u and H_d are contained in $\mathbf{10}_\Phi$. Then the Yukawa matrices for up- and down-quark can be written as

$$Y_u = Y_\Phi, \quad Y_d = Y_\Phi U_{16}^0. \quad (2.14)$$

Without loss of generality, we can take the diagonal matrix \hat{Y}_Φ as Y_Φ . Let us consider a situation that the three massless modes of $\mathbf{5}$ fields are mainly composed of $\mathbf{5}_1$, $\mathbf{5}'_1$ and $\mathbf{5}_2$. As noted before, the two $\mathbf{5}$ fields from the third generation field tend to be become superheavy and therefore it is reasonable. In such a case, by solving Eq. (2.11), U_{10}^0 and U_{16}^0 are approximately obtained as

$$U_{10}^0 \sim \begin{pmatrix} 0 & 1 & 0 \\ p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \end{pmatrix}, \quad U_{16}^0 \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ r_1 & r_2 & r_3 \end{pmatrix} \quad (2.15)$$

where $p_i, q_i, r_i \ll 1$ and their values are determined by Eq. (2.11) (see Appendix B). For example, when

$$\hat{Y}_\Phi \sim \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Y_C \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad (2.16)$$

and $r \sim \lambda^{0.5}$, Eq. (2.11) becomes

$$U_{10}^0 = \begin{pmatrix} c_{11}\lambda^{0.5} & c_{21}\lambda^{-0.5} & c_{31}\lambda^{-2.5} \\ c_{12}\lambda^{1.5} & c_{22}\lambda^{0.5} & c_{32}\lambda^{-1.5} \\ c_{13}\lambda^{3.5} & c_{23}\lambda^{2.5} & c_{33}\lambda^{0.5} \end{pmatrix} U_{16}^0, \quad (2.17)$$

where c_{ij} are $\mathcal{O}(1)$ coefficients. Taking account of the unitarity of U , U_{10}^0 and U_{16}^0 are calculated at the leading order as

$$U_{10}^0 \sim \begin{pmatrix} \mathcal{O}(\lambda^{2.5}) & 1 & \mathcal{O}(\lambda^{1.5}) \\ (c_{12} - \frac{c_{11}c_{32}}{c_{31}})\lambda^{1.5} & \frac{c_{32}}{c_{31}}\lambda & (c_{22} - \frac{c_{21}c_{32}}{c_{31}})\lambda^{0.5} \\ (c_{13} - \frac{c_{11}c_{33}}{c_{31}})\lambda^{3.5} & \frac{c_{33}}{c_{31}}\lambda^3 & (c_{23} - \frac{c_{21}c_{33}}{c_{31}})\lambda^{2.5} \end{pmatrix}, \quad (2.18)$$

$$U_{16}^0 \sim \begin{pmatrix} 1 & 0 & 0 \\ \mathcal{O}(\lambda^2) & 0 & 1 \\ -\frac{c_{11}}{c_{31}}\lambda^3 & -\frac{1}{c_{31}}\lambda^{2.5} & -\frac{c_{21}}{c_{31}}\lambda^2 \end{pmatrix}, \quad (2.19)$$

where three zero components are rotated away by the 3×3 unitary matrix from the right. Then, we can obtain Y_d as

$$Y_d \sim \begin{pmatrix} \lambda^6 & 0 & 0 \\ \mathcal{O}(\lambda^6) & 0 & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix} \quad (2.20)$$

Since $Y_u = \hat{Y}_\Phi$ is diagonal, the $(1, 3)$ and $(1, 2)$ components of the CKM matrix are quite small in this setup. Note that the situation does not change so much if the main part of $\bar{\mathbf{5}}_3^0$ is $\bar{\mathbf{5}}'_1$. When the main part of $\bar{\mathbf{5}}_3^0$ is $\bar{\mathbf{5}}'_1$, all the CKM mixings become quite small.

There are basically two ways to avoid the smallness of the mixing angles of the CKM matrix in E_6 models. One way is to make the other Yukawa couplings like Y_C contribute the Yukawa couplings, Y_u and Y_d by, for example, introducing the mixings of $\mathbf{27}_C$ into the MSSM Higgs H_u and/or H_d . This is quite reasonable because we have $\mathbf{27}_C$ to break $SO(10)$ into $SU(5)$ in the E_6 GUT models, though it must be considered how to obtain the mixing of $\mathbf{27}_C$ in solving the doublet-triplet splitting problem. The other way is to break the E_6 GUT relation for the Yukawa couplings Y_Φ by, for example, introducing higher dimensional interactions including E_6 Higgses which break E_6 into the SM gauge group. To avoid the undesirable $SU(5)$ GUT relation $Y_d = Y_e^T$, we must introduce such higher dimensional interactions including adjoint Higgs field A such as

$$W \supset \frac{1}{\Lambda} \Psi_i (A \Psi_j) H \quad (H = \Phi, C) \quad (2.21)$$

with a cutoff scale Λ . Then after developing the VEV of A , the coefficients of the Yukawa interactions do not respect $SU(5)$ symmetry though generically the contributions from such higher dimensional interactions are suppressed because the factor $\langle A \rangle / \Lambda$ is much smaller than one. Generically, it is not easy that the contributions of such higher dimensional interactions to the Yukawa couplings become sizable. One of the easiest way to obtain sizable contributions of such higher dimensional interactions is to introduce the anomalous $U(1)_A$ gauge symmetry [37]. Because the VEVs of fields Z_i with $U(1)_A$ charges z_i can be obtained as

$$\langle Z_i \rangle \sim \begin{cases} 0 & (z_i > 0) \\ (\xi/\Lambda)^{-z_i} & (z_i < 0) \end{cases}, \quad (2.22)$$

the higher dimensional interactions have the same order contributions as the usual Yukawa interactions at tree level. Here, ξ is the parameter of the Fayet-Iliopoulos D -term. Another way is to introduce family symmetry which forbids some of tree Yukawa interactions and allows the higher dimensional interactions. In next section, we discuss a concrete example in which such family symmetry is introduced.

3 CKM matrix in model with horizontal $SU(2)_H$ symmetry

In this section, we construct a simple model of flavor in E_6 GUT and investigate the structure of CKM matrix in this model. Here we focus on a model with a family symmetry $SU(2)_H$ and spontaneous CP violation. Such a kind of model is examined in Ref. [26] and we consider a model which has the important features of the model.

First of all, we introduce a pair of $SU(2)_H$ doublet fields F_a and \bar{F}^a which is responsible for the breaking of the family symmetry, $SU(2)_H$. Here the indices a represent the

	Ψ_a	Ψ_3	F_a	\bar{F}^a	Φ	$\bar{\Phi}$	C	\bar{C}	A
E_6	27	27	1	1	27	27	27	27	78
$SU(2)_H$	2	1	2	2	1	1	1	1	1
Z_3	0	0	1	0	0	0	2	0	0

Table 1: Field contents and charge assignment under $E_6 \otimes SU(2)_H \otimes Z_3$

transformation property in $SU(2)_H$ and it can be raised or lowered by means of the anti-symmetric symbols ϵ^{ab} and ϵ_{ab} . Taking into account the D -flatness condition and $SU(2)_H$ gauge degree of freedom, the VEV of F_a and \bar{F}^a can be generally written as

$$\langle F \rangle = \begin{pmatrix} 0 \\ v_F e^{i\delta} \end{pmatrix}, \quad \langle \bar{F} \rangle = \begin{pmatrix} 0 \\ v_F \end{pmatrix} \quad (3.1)$$

where we suppose that there appears nonzero phase δ and it breaks the CP invariance of the theory spontaneously. As pointed out in Ref. [25], spontaneous CP violation can be a natural solution of the SUSY CP problem which may arise in considering the effect of SUSY breaking. That is, a real up-quark Yukawa matrix is favored for model building because the non-universality of the up-squark mass with complex Yukawa couplings leads to the problem of chromo-EDM constraint [27]. To accomplish a real up-quark sector, we introduce a Z_3 discrete symmetry and impose charge assignment shown in Table 1. This Z_3 symmetry gives further advantage for the SUSY CP problem, because it prohibits dangerous terms of Kähler potential (e.g., $K \supset \tilde{m}^2 \theta^2 \bar{\theta}^2 \Psi_3^\dagger (\Psi_a F^a)$) which induce complex squark masses.

With the above setup, we can write down the interaction terms which contribute to the Yukawa matrices:

$$Y_\Phi : \begin{pmatrix} 0 & \Psi_a (A \Psi^a) & 0 \\ \Psi_a (A \Psi^a) & (\Psi_a \bar{F}^a)^2 & (\Psi_a \bar{F}^a) \Psi_3 \\ 0 & (\Psi_a \bar{F}^a) \Psi_3 & \Psi_3 \Psi_3 \end{pmatrix} \Phi, \quad (3.2)$$

$$Y_C : \begin{pmatrix} 0 & (\Psi_a F^a) (\Psi_b \bar{F}^b) & (\Psi_a F^a) \Psi_3 \\ (\Psi_a F^a) (\Psi_b \bar{F}^b) & 0 & 0 \\ (\Psi_a F^a) \Psi_3 & 0 & 0 \end{pmatrix} C \quad (3.3)$$

where we adopt terms which give the leading contribution and the higher dimensional terms are neglected. In this paper, we often take the unit in which $\Lambda = 1$. Note that the terms like $\epsilon^{ab} \Psi_a \Psi_b \Phi$ do not exist because E_6 singlet can be formed from the totally symmetric combination of three **27**s.

In the following discussion, we assume that the MSSM Higgs fields H_u and H_d are

$$H_u \simeq \mathbf{10}_\Phi [\mathbf{5}_\Phi], \quad H_d \simeq \mathbf{10}_\Phi [\bar{\mathbf{5}}'_\Phi] + \alpha \mathbf{16}_C [\bar{\mathbf{5}}_C] + \beta \mathbf{10}_C [\bar{\mathbf{5}}'_C] \quad (3.4)$$

with $\alpha, \beta \ll 1$. Here, there is no component from C in H_u . This is important in order to obtain real Yukawa matrix for up-type quarks. On the other hand, $\alpha \neq 0$ or $\beta \neq 0$ is required to obtain complex Yukawa couplings for down-type quarks, which is important to obtain non vanishing KM phase. Now we write down the explicit form of the up-quark Yukawa matrix Y_u as follows:

$$Y_u = Y_\Phi^{(u)} \equiv \begin{pmatrix} 0 & d & 0 \\ -d & c & b \\ 0 & b & a \end{pmatrix} \sim \begin{pmatrix} 0 & \lambda^A & 0 \\ \lambda^A & \lambda^{2F} & \lambda^F \\ 0 & \lambda^F & 1 \end{pmatrix}. \quad (3.5)$$

where a, b, c, d are real parameters. Here the right hand side of Eq. (3.5) indicates the order of each element and we assume $\lambda^A \ll \lambda^{2F} \ll 1$. Then Y_u can be diagonalized by unitary transformations of up-type quark fields as $V_{uL}^\dagger Y_u V_{uR} = \text{diag}(y_u, y_c, y_t)$. The leading order contribution of V_{uL} and V_{uR} can be calculated to be

$$V_{uL} \simeq \begin{pmatrix} 1 & \frac{ad}{ac-b^2} & 0 \\ -\frac{ad}{ac-b^2} & 1 & \frac{b}{a} \\ \frac{bd}{ac-b^2} & -\frac{b}{a} & 1 \end{pmatrix}, \quad V_{uR} \simeq \begin{pmatrix} 1 & -\frac{ad}{ac-b^2} & 0 \\ \frac{ad}{ac-b^2} & 1 & \frac{b}{a} \\ -\frac{bd}{ac-b^2} & -\frac{b}{a} & 1 \end{pmatrix} \quad (3.6)$$

and the Yukawa couplings $y_f \equiv m_f/v_u$ ($f = u, c, t$) are obtained as

$$y_u \simeq \frac{ad^2}{ac-b^2}, \quad y_c \simeq \frac{ac-b^2}{a}, \quad y_t \simeq a. \quad (3.7)$$

(See appendix A for this approximation.)

The structure of the down-quark sector is a little complicated because of the mixing of $\bar{\mathbf{5}}$ fields shown in Eq. (2.9). Since the contribution from massless modes $\bar{\mathbf{5}}_i^0$ can be extracted in a manner shown in Eq. (2.13), down-quark Yukawa matrix is

$$Y_d = Y_\Phi^{(d)} U_{\mathbf{16}}^0 + \alpha Y_C U_{\mathbf{10}}^0 + \beta Y_C U_{\mathbf{16}}^0 \quad (3.8)$$

where $Y_\Phi^{(d)} \neq Y_\Phi^{(u)}$ as mentioned in the previous section. That is, terms involving an adjoint Higgs A give different contribution for each component of the term:

$$\Psi_a(\langle A \rangle \Psi^a) \Phi \ni (v_{\bar{u}} - v_Q) \epsilon^{ab} Q_a \bar{u}_b H_u + (v_{\bar{d}} - v_Q) \epsilon^{ab} Q_a \bar{d}_b H_d \quad (3.9)$$

where $\langle A \rangle \psi \equiv v_\psi \psi$ for a component field ψ . Thus, when $\Psi_a(A \Psi^a) \Phi$ gives a term $d Q_1 \bar{u}_2 H_u$, there is a term $-(24\epsilon + 1)/5 \cdot d Q_1 \bar{d}_2 H_d$ for down-quark sector, where $\epsilon \equiv y/z$ with y and z are defined by Eq. (2.4). Since we have defined $(Y_\Phi^{(u)})_{12} = -(Y_\Phi^{(u)})_{21} \equiv d$, it leads to be $(Y_\Phi^{(d)})_{12} = -(Y_\Phi^{(d)})_{21} = -(24\epsilon + 1)d/5$. Moreover, the effect of an adjoint Higgs also appears on the mass term between $\mathbf{5}$ and $\bar{\mathbf{5}}$ fields as

$$\Psi_a(\langle A \rangle \Psi^a) \langle \Phi \rangle \ni (v_{\bar{d}'} - v_d) \epsilon^{ab} d_a \bar{d}_b' \langle \mathbf{1}_\Phi \rangle. \quad (3.10)$$

Therefore we must use $M_{d\bar{d}} \equiv (Y_\Phi^{(M_{d\bar{d}})}, r Y_C) \langle \Phi \rangle$ with $(Y_\Phi^{(M_{d\bar{d}})})_{12} = (Y_\Phi^{(M_{d\bar{d}})})_{21} = (24\epsilon - 4)d/5$ instead of M in Eq. (2.7). Taking them into account, one can write down the specific form of the down-quark Yukawa matrix. For this purpose, let us express the component of Y_C as

$$Y_C \equiv \begin{pmatrix} 0 & f & e \\ f & 0 & 0 \\ e & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & \lambda^{2F} & \lambda^F \\ \lambda^{2F} & 0 & 0 \\ \lambda^F & 0 & 0 \end{pmatrix} \quad (3.11)$$

where $\arg(e) = \arg(f) = \delta \neq 0$ and right-hand side of equation represents the order of magnitude for each component. If the main modes of $\bar{\mathbf{5}}_1^0$, $\bar{\mathbf{5}}_2^0$ and $\bar{\mathbf{5}}_3^0$ are $\bar{\mathbf{5}}_1$, $\bar{\mathbf{5}}_1'$ and $\bar{\mathbf{5}}_2$,[‡]

[‡] If the main modes $\bar{\mathbf{5}}_1^0$, $\bar{\mathbf{5}}_2^0$ and $\bar{\mathbf{5}}_3^0$ are $\bar{\mathbf{5}}_1$, $\bar{\mathbf{5}}_2$ and $\bar{\mathbf{5}}_1'$, respectively, then, the V_{cb} is vanishing at leading order unless $\alpha \sim \mathcal{O}(1)$. If $\alpha \sim \mathcal{O}(1)$, generically, no cancellation happens.

respectively, then we can calculate Y_d given by (3.8) to be

$$Y_d \simeq \begin{pmatrix} -\alpha r \frac{af^2+ce^2-2bef}{ac-b^2} - \beta \frac{24\epsilon-4}{5} \frac{d(be-af)}{ac-b^2} & -\alpha \frac{24\epsilon-4}{5} \frac{d(be-af)}{ac-b^2} - \beta \left(\frac{24\epsilon-4}{5}\right)^2 \frac{ad^2}{r(ac-b^2)} & -\frac{24\epsilon+1}{5} d \\ \frac{24\epsilon+1}{5} d - \frac{24\epsilon-4}{5} \frac{bd(be-af)}{e(ac-b^2)} + \beta f & -\left(\frac{24\epsilon-4}{5}\right)^2 \frac{abd^2}{re(ac-b^2)} + \alpha f & \frac{ce-bf}{e} \\ -\frac{24\epsilon-4}{5} \frac{ad(be-af)}{e(ac-b^2)} + \beta e & -\left(\frac{24\epsilon-4}{5}\right)^2 \frac{a^2d^2}{re(ac-b^2)} + \alpha e & \frac{be-af}{e} \end{pmatrix} \\ \sim \begin{pmatrix} \alpha r \lambda^{2F} + \beta \lambda^A & \alpha \lambda^A + \beta r^{-1} \lambda^{2A-2F} & \lambda^A \\ \lambda^A + \beta \lambda^{2F} & r^{-1} \lambda^{2A-2F} + \alpha \lambda^{2F} & \lambda^{2F} \\ \lambda^{A-F} + \beta \lambda^F & r^{-1} \lambda^{2A-3F} + \alpha \lambda^F & \lambda^F \end{pmatrix}. \quad (3.12)$$

Since $\lambda^A \ll \lambda^{2F} \ll 1$ is supposed, Y_d has a hierarchical structure $(Y_d)_{ij} \ll (Y_d)_{kj}$, $(Y_d)_{ij} \ll (Y_d)_{il}$ with $i < k$, $j < l$ for a proper choice of α and r . So the unitary matrix which diagonalizes Y_d as $V_{dL}^\dagger Y_d V_{dR} = \text{diag}(y_d, y_s, y_b)$ is

$$V_{dL} \simeq \begin{pmatrix} 1 & s_3^{dL*} & -\frac{24\epsilon+1}{5} \frac{de}{be-af} \\ -s_3^{dL} & 1 & \frac{ce-bf}{be-af} \\ \frac{1}{be-af} \left[\frac{24\epsilon+1}{5} de + s_3^{dL} (ce-bf) \right] & -\frac{ce-bf}{be-af} & 1 \end{pmatrix} \quad (3.13)$$

at leading order. Here $s_3^{dL} \equiv \sin \theta_3^{dL} e^{i\chi_3^{dL}}$ is defined in Appendix A, and approximately given by

$$s_3^{dL} \simeq \frac{(Y_d)_{12}(Y_d)_{33} - (Y_d)_{13}(Y_d)_{32}}{(Y_d)_{22}(Y_d)_{33} - (Y_d)_{23}(Y_d)_{32}}. \quad (3.14)$$

Now we can calculate the CKM matrix element defined through $V_{\text{CKM}} \equiv V_{uL}^\dagger V_{dL}$ using the explicit formulas Eqs. (3.6) and (3.13):

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 & s_3^{dL*} - \frac{ad}{ac-b^2} & -\frac{24\epsilon+6}{5} \frac{de}{be-af} \\ -V_{us}^* & 1 & \frac{e(ac-b^2)}{a(be-af)} \\ V_{ts}V_{cd} - V_{ub} & -V_{cb} & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda^{A-2F} & \lambda^{A-F} \\ \lambda^{A-2F} & 1 & \lambda^F \\ \lambda^{A-F} & \lambda^F & 1 \end{pmatrix}. \quad (3.15)$$

To obtain a realistic flavor structure, we assume that $\lambda^A \sim \lambda^5$, $\lambda^F \sim \lambda^2$, $\alpha \sim \lambda^{0.5}$, $\beta \sim \lambda$ and $r \sim \lambda^{1.5}$, which lead to

$$Y_u \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}. \quad (3.16)$$

If the VEV of the adjoint Higgs $\mathbf{45}_A$ is proportional to the generator of $U(1)_{B-L}$, i.e., $\epsilon = -1/4$, the leading contribution to V_{ub} vanishes. (We discuss more general examples in which such cancellation happens only when $U(1)_{B-L}$ is not broken by the adjoint Higgs VEV in appendix B.) The next-to-leading contribution to V_{ub} can be estimated to be

$$V_{ub} \simeq \frac{(Y_d)_{32}}{(Y_d)_{33}^2} \left[\frac{ad}{ac-b^2} (Y_d)_{22} e^{i\phi} + (Y_d)_{12} \right] \sim \mathcal{O}(\lambda^4), \quad (3.17)$$

where $\phi = -2 \arg((Y_d)_{32}/(Y_d)_{33})$. Therefore, we can obtain as

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (3.18)$$

Since the measured value for V_{ub} is $\mathcal{O}(\lambda^4)$ rather than $\mathcal{O}(\lambda^3)$, this result indicates that the direction of the adjoint Higgs VEV is proportional to $U(1)_{B-L}$ in this E_6 GUT model. On the other hand, this direction plays an important role in solving the doublet-triplet splitting problem by Dimopoulos-Wilczek mechanism [34, 35]. This coincidence is quite interesting and suggestive.

4 Summary

In this paper, we have examined how to obtain the CKM matrix as in Eq. (3.18) in an E_6 GUT with a family symmetry $SU(2)_H$. Especially we have studied the reason for the cancellation by which V_{ub} becomes $\mathcal{O}(\lambda^4)$, not $\mathcal{O}(\lambda^3)$. Since the measured value of V_{ub} is $\mathcal{O}(\lambda^4)$, it must be valuable to know the reason for the cancellation. First, we have shown that in a simple E_6 GUT model, the Cabibbo mixing and V_{ub} tend to be much smaller than the naively expected values because of the E_6 Yukawa unification. Of course, in order to obtain sizable Cabibbo mixing, we must avoid such suppression for the Cabibbo mixing. Therefore, we have studied several ways to avoid such suppression of the CKM mixings. Generically, when we introduce something to avoid the suppression, it spoils the suppression of V_{ub} . Actually, in the E_6 model with a family symmetry and spontaneous CP violation, generically, the suppression of V_{ub} is spoiled. Only when the adjoint VEV does not break $U(1)_{B-L}$, the cancellation happens. Since such a direction of the VEV plays an important role in solving the doublet-triplet splitting problem by Dimopoulos-Wilczek mechanism, it is interesting that the measured value of V_{ub} is of order λ^4 . It may be concluded that the experiments measured the direction of the adjoint VEV in this E_6 GUT model with $SU(2)_H$ by measuring the size of V_{ub} . Since the direct search of the GUT is very difficult, indirect measurements like this must be important. Though one indirect measurement is not sufficient to confirm a scenario, we hope that various indirect measurements will confirm some GUT scenario in future.

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A Diagonalization procedure of the hierarchical matrix

In this appendix we summarize the procedure of diagonalization of the 3×3 matrix Y which is needed in computing the CKM matrix. Here we focus on the case where the matrix Y has a hierarchical structure $Y_{ij} \ll Y_{kj}$ and $Y_{ij} \ll Y_{il}$ with $i < k$ and $j < l$. We mainly follow the procedure given in Ref. [40].

The arbitrary matrix Y can be diagonalized multiplying the unitary matrices V_L and

V^R as $V_L^\top Y V_R = Y^D$, where Y^D is a diagonal matrix. We can parametrize V_L and V_R so that

$$V_L^\top \equiv \begin{pmatrix} c_3^L & -s_3^L & 0 \\ s_3^{L*} & c_3^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2^L & 0 & -s_2^L \\ 0 & 1 & 0 \\ s_2^{L*} & 0 & c_2^L \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1^L & -s_1^L \\ 0 & s_1^{L*} & c_1^L \end{pmatrix} \equiv P_3^L P_2^L P_1^L, \quad (\text{A.1})$$

$$V_R \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1^R & s_1^R \\ 0 & -s_1^{R*} & c_1^R \end{pmatrix} \begin{pmatrix} c_2^R & 0 & s_2^R \\ 0 & 1 & 0 \\ -s_2^{R*} & 0 & c_2^R \end{pmatrix} \begin{pmatrix} c_3^R & s_3^R & 0 \\ -s_3^{R*} & c_3^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv P_1^{R\dagger} P_2^{R\dagger} P_3^{R\dagger} \quad (\text{A.2})$$

where $s_i^{L/R} \equiv \sin \theta_i^{L/R} e^{i\chi_i^{L/R}}$ and $c_i^{L/R} \equiv \cos \theta_i^{L/R}$ with $i = 1, 2, 3$. If we write the components of Y as

$$Y \equiv \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}, \quad (\text{A.3})$$

the procedure of diagonalization is expressed as follows:

$$\begin{aligned} V_L^\top Y V_R &= P_3^L P_2^L P_1^L \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix} P_1^{R\dagger} P_2^{R\dagger} P_3^{R\dagger} \simeq P_3^L P_2^L \begin{pmatrix} y_{11} & y'_{12} & y_{13} \\ y'_{21} & y'_{22} & 0 \\ y_{31} & 0 & y_{33} \end{pmatrix} P_2^{R\dagger} P_3^{R\dagger} \\ &\simeq P_3^L \begin{pmatrix} y'_{11} & y'_{12} & 0 \\ y'_{21} & y'_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix} P_3^{R\dagger} = \begin{pmatrix} y''_{11} & 0 & 0 \\ 0 & y'_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix} \end{aligned} \quad (\text{A.4})$$

Here we supposed $|s_i^{L/R}| \ll 1$ and $c_i^{L/R} \simeq 1$ and

$$y'_{22} \simeq y_{22} - \frac{y_{23}y_{32}}{y_{33}}, \quad y'_{12} \simeq y_{12} - \frac{y_{13}y_{32}}{y_{33}}, \quad y'_{21} \simeq y_{21} - \frac{y_{23}y_{31}}{y_{33}}, \quad (\text{A.5})$$

$$y'_{11} \simeq y_{11} - \frac{y_{13}y_{31}}{y_{33}}, \quad y''_{11} \simeq y'_{11} - \frac{y'_{12}y'_{21}}{y'_{22}} \quad (\text{A.6})$$

which are good approximation if the hierarchy of the matrix Y is realized sufficiently. Then the mixing angles for the left-handed field are

$$s_1^L \simeq \frac{y_{23}}{y_{33}}, \quad s_2^L \simeq \frac{y_{13}}{y_{33}}, \quad s_3^L \simeq \frac{y_{12}y_{33} - y_{13}y_{32}}{y_{22}y_{33} - y_{23}y_{32}}. \quad (\text{A.7})$$

Note that, if we consider the diagonalization of 2×2 matrix

$$\begin{pmatrix} c_L & -s_L \\ s_L^* & c_L \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} c_R & s_R \\ -s_R^* & c_R \end{pmatrix} = \begin{pmatrix} m'_{11} & 0 \\ 0 & m'_{22} \end{pmatrix} \quad (\text{A.8})$$

where $s_{L/R} \equiv \sin \theta_{L/R} e^{i\chi_{L/R}}$ and $c_{L/R} \equiv \cos \theta_{L/R}$, the explicit formula for $\tan 2\theta_{L/R}$ can be obtained as

$$\tan 2\theta_L = \frac{2(m_{12}m_{22} + m_{11}m_{21}e^{2i\chi_R})}{m_{22}^2 e^{i\chi_L} - m_{11}^2 e^{-i(\chi_L - 2\chi_R)} + m_{21}^2 e^{i(\chi_L + 2\chi_R)} - m_{12}^2 e^{-i\chi_L}}, \quad (\text{A.9})$$

$$\tan 2\theta_R = \frac{2(m_{11}m_{12} + m_{21}m_{22}e^{2i\chi_L})}{m_{22}^2 e^{-i(\chi_R - 2\chi_L)} - m_{11}^2 e^{i\chi_R} - m_{21}^2 e^{i(\chi_R + 2\chi_L)} + m_{12}^2 e^{-i\chi_R}}. \quad (\text{A.10})$$

Here $\chi_{L/R}$ are determined so that the right-hand side of each equation becomes real.

Therefore the above approximation is correct if $m_{22} \gg m_{11}, m_{12}, m_{21}$ in Eq. (A.9).

We now write down the explicit expression for the CKM matrix elements using the above parametrization. Since the rotation matrix for left-handed up- and down-quarks are

$$V_{u/dL}^\top = P_3^{u/dL} P_2^{u/dL} P_1^{u/dL} \simeq \begin{pmatrix} 1 & -s_3^{u/dL} & -s_2^{u/dL} + s_1^{u/dL} s_3^{u/dL} \\ s_3^{u/dL*} & 1 & -s_1^{u/dL} \\ s_2^{u/dL*} & s_1^{u/dL*} & 1 \end{pmatrix} \quad (\text{A.11})$$

in this parametrization, CKM matrix element defined by $V_{\text{CKM}} \equiv V_{uL}^\dagger V_{dL}$ can be obtained as

$$V_{\text{CKM}}^* \equiv V_{uL}^\top (V_{dL}^\top)^\dagger \simeq \begin{pmatrix} 1 & s_3^{dL} - s_3^{uL} & s_2^{dL} - s_2^{uL} - s_3^{uL}(s_1^{dL} - s_1^{uL}) \\ -V_{us}^* & 1 & s_1^{dL} - s_1^{uL} \\ V_{us}^* V_{cb}^* - V_{ub}^* & -V_{cb}^* & 1 \end{pmatrix}. \quad (\text{A.12})$$

In Eqs. (A.11), we supposed $s_2^{L/R} \ll s_1^{L/R}, s_3^{L/R}$ as can be expected from the experimental value for the CKM matrix elements.

B Structure of CKM matrix in E_6 GUT

In this appendix, first, we give an explicit form of U_{10}^0 and U_{16}^0 when higher dimensional interactions are neglected. Next, we show that the cancellation in V_{ub} at the leading order more generally happens when $U(1)_{B-L}$ is not broken by the adjoint VEV.

First we consider the superpotential given in Eq. (2.6) neglecting the higher dimensional terms which involve the VEV of some fields. In such a case, we can always adopt the basis of the matter fields Ψ_i where Y_Φ is a diagonal matrix. Then we express Y_Φ and Y_C to be

$$Y_\Phi \equiv \begin{pmatrix} a_1 & & \\ & a_2 & \\ & & a_3 \end{pmatrix}, \quad Y_C \equiv \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad (\text{B.1})$$

in this basis. U_{16}^0 is defined in Eq. (2.9) and represents the mixing of $\bar{\mathbf{5}}$ fields for the massless modes $\bar{\mathbf{5}}_i^0$. Then the mixing matrices U_{10}^0 and U_{16}^0 can be calculated through Eq. (2.11) and expressed as

$$U_{10}^0 \simeq \begin{pmatrix} 0 & 1 & 0 \\ r(b_{23}b_{11} - b_{21}b_{13})/a_2b_{13} & a_1b_{23}/a_2b_{13} & r(b_{23}b_{12} - b_{22}b_{13})/a_2b_{13} \\ r(b_{33}b_{11} - b_{31}b_{13})/a_3b_{13} & a_1b_{33}/a_3b_{13} & r(b_{33}b_{12} - b_{32}b_{13})/a_3b_{13} \end{pmatrix}, \quad (\text{B.2})$$

$$U_{16}^0 \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -b_{11}/b_{13} & -a_1/rb_{13} & -b_{12}/b_{13} \end{pmatrix} \quad (\text{B.3})$$

Here we assume that the massless modes of $\bar{\mathbf{5}}$ fields $\bar{\mathbf{5}}_1^0, \bar{\mathbf{5}}_2^0, \bar{\mathbf{5}}_3^0$ correspond to $\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_1', \bar{\mathbf{5}}_2$, respectively, and $(U_{10}^0)_{2i}, (U_{10}^0)_{3i}, (U_{16}^0)_{3i} \ll 1$ with $i = 1, 2, 3$.

Next we consider the more general situation where the higher dimensional interaction terms also contribute to the Yukawa matrices. As shown in Section 2, we must take into account the fact that the VEV of an adjoint Higgs gives the different contribution for the

up-quark sector and down-quark sector. Therefore we should express the Yukawa matrices as

$$Y_u = Y_\Phi^{(u)}, \quad Y_d = Y_\Phi^{(d)} U_{\mathbf{16}}^0 \quad (\text{B.4})$$

when $H_u \simeq \mathbf{10}_\Phi[\mathbf{5}_\Phi]$ and $H_d \simeq \mathbf{10}_\Phi[\bar{\mathbf{5}}'_\Phi]$. The effective Yukawa matrices $Y_\Phi^{(u)}$ and $Y_\Phi^{(d)}$ are not equal in general. Moreover, $Y_\Phi^{(u)}$ and $Y_\Phi^{(d)}$ are no longer guaranteed to be symmetric matrices, and we cannot always make them diagonal by the redefinition of the matter fields Ψ_i . So the general expressions for $Y_\Phi^{(u)}$ and $Y_\Phi^{(d)}$ are

$$Y_\Phi^{(u)} \equiv \begin{pmatrix} a_{11}^u & a_{12}^u & a_{13}^u \\ a_{21}^u & a_{22}^u & a_{23}^u \\ a_{31}^u & a_{32}^u & a_{33}^u \end{pmatrix}, \quad Y_\Phi^{(d)} \equiv \begin{pmatrix} a_{11}^d & a_{12}^d & a_{13}^d \\ a_{21}^d & a_{22}^d & a_{23}^d \\ a_{31}^d & a_{32}^d & a_{33}^d \end{pmatrix}. \quad (\text{B.5})$$

Since the mass terms between $\mathbf{5}$ and $\bar{\mathbf{5}}$ matter fields also receive the different contribution from the higher dimensional terms, $M \equiv (Y_\Phi^{(M)}, rY_C) \langle \Phi \rangle$ with $Y_\Phi^{(M)}$ different from $Y_\Phi^{(u)}$ and $Y_\Phi^{(d)}$.

Although it is difficult to derive the expression of CKM matrix in such a complicated case, we can show the characteristic feature for some components of the CKM matrix. To see this, we focus on the third column of the mixing matrix $U_{\mathbf{16}}^0$:

$$U_{\mathbf{16}}^0 \equiv \begin{pmatrix} \star & \star & x_1 \\ \star & \star & x_2 \\ \star & \star & x_3 \end{pmatrix} \quad (\text{B.6})$$

where $x_1 \simeq 0$ and $x_2 \simeq 1$ if we assume that $\bar{\mathbf{5}}_1^0, \bar{\mathbf{5}}_2^0, \bar{\mathbf{5}}_3^0$ are mainly composed of $\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2$ respectively. Then the Yukawa matrix for the down-quark is

$$Y_d = Y_\Phi^{(d)} U_{\mathbf{16}}^0 = \begin{pmatrix} \star & \star & a_{11}^d x_1 + a_{12}^d x_2 + a_{13}^d x_3 \\ \star & \star & a_{21}^d x_1 + a_{22}^d x_2 + a_{23}^d x_3 \\ \star & \star & a_{31}^d x_1 + a_{32}^d x_2 + a_{33}^d x_3 \end{pmatrix}. \quad (\text{B.7})$$

From these information, we can see from Eq. (A.7) that

$$s_1^{uL} \simeq \frac{a_{23}^u}{a_{33}^u}, \quad s_2^{uL} \simeq \frac{a_{13}^u}{a_{33}^u}, \quad s_3^{uL} \simeq \frac{a_{12}^u a_{33}^u - a_{23}^u a_{13}^u}{a_{22}^u a_{33}^u - a_{23}^u a_{32}^u}, \quad (\text{B.8})$$

$$s_1^{dL} \simeq \frac{a_{21}^d x_1 + a_{22}^d x_2 + a_{23}^d x_3}{a_{31}^d x_1 + a_{32}^d x_2 + a_{33}^d x_3}, \quad s_2^{dL} \simeq \frac{a_{11}^d x_1 + a_{12}^d x_2 + a_{13}^d x_3}{a_{31}^d x_1 + a_{32}^d x_2 + a_{33}^d x_3}, \quad (\text{B.9})$$

if Y_u and Y_d have a hierarchical structure which is supposed in Appendix A. Now we can obtain the specific expression for V_{ub} from Eq. (A.12):

$$V_{ub}^* \simeq s_2^{dL} - s_2^{uL} - s_3^{uL} (s_1^{dL} - s_1^{uL}). \quad (\text{B.10})$$

If we take $a_{ij}^u = a_{ij}^d \equiv a_{ij}$ in Eqs. (B.8) and (B.9), Eq. (B.10) can be rewritten as

$$V_{ub}^* \simeq \frac{x_1}{a_{31}x_1 + a_{32}x_2 + a_{33}x_3} \left[a_{11} + \frac{a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})}{a_{22}a_{33} - a_{23}a_{32}} \right] \quad (\text{B.11})$$

and $V_{ub} \propto x_1$ is derived. Since $x_1 \simeq 0$ in our setup, the leading order contribution for V_{ub} vanishes if $a_{ij}^u = a_{ij}^d$ is satisfied. On the other hand, if $a_{ij}^u \neq a_{ij}^d$, the non-vanishing contribution to V_{ub} generally appears. For example, let us assume that $a_{12}^u \neq a_{12}^d$ and $a_{21}^u \neq$

a_{21}^d , which can be occurred by the presence of the term $\Psi_a(A\Psi^a)\Phi$. Here Ψ_a is a horizontal $SU(2)_H$ doublet matter field which appears in Section 3. Then, the additional contribution to V_{ub} is

$$\delta V_{ub}^* = -\frac{(a_{12}^u - a_{12}^d)}{(a_{31}x_1 + a_{32}x_2 + a_{33}x_3)}x_2 \quad (\text{B.12})$$

and it gives non-vanishing contribution because $x_2 \simeq 1$ if the third generation of the massless $\bar{\mathbf{5}}$ field $\bar{\mathbf{5}}_3^0$ is mainly $\bar{\mathbf{5}}_2$. This is the case of Eq. (3.15) and this contribution vanishes if $a_{12}^u = a_{12}^d$ is satisfied. Note that if the VEV of the adjoint Higgs field does not break $U(1)_{B-L}$, then $a_{ij}^u = a_{ij}^d$ is satisfied and V_{ub} is vanishing at leading order. The size of V_{ub} at sub-leading order is dependent on the explicit model.

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